

An Optimization-Based Simultaneous Approach to the Determination of Muscular, Ligamentous, and Joint Contact Forces Provides Insight into Musculoligamentous Interaction

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Abstract

Typical inverse dynamics approaches to the calculation of muscle, ligament and joint contact forces are based upon a step-wise solution of the equations of motion. This approach is therefore limited in its ability to provide insight as to the muscular, ligamentous and articular interactions that create joint stability. In this study, a new musculoskeletal model of the lower limb is described, in which the equations of motion describing the force and moment equilibrium at the joints of the lower limb are solved simultaneously using optimization techniques. The new model was employed to analyse vertical jumping using a variety of different optimization cost functions and the results were compared to more traditional approaches. The new model was able to find a solution with lower muscular force upper bounds due to the ability of the ligaments to contribute to moment equilibrium at the ankle and knee joints. Equally, the new model produced lower joint contact forces than traditional approaches for cases which also included a consideration as to ligament or joint contact forces within the cost function. This study demonstrates the possibility of solving the inverse dynamic equations of motion simultaneously using contemporary technology, and further suggests that this might be important due to the complementary function of the muscles and ligaments in creating joint stability.

Keywords: musculoskeletal modelling, muscle force, joint contact force, ligament force, inverse dynamics

Introduction

Inverse dynamics analyses have been widely employed within bioengineering to improve the understanding of movement⁴⁰. To this end, the combination of inverse dynamics and optimization techniques has been used to calculate inter-segmental forces and moments, muscle forces, joint reaction forces and ligament forces⁴⁰. In particular, the traditional approach to calculating the kinematic and kinetic quantities of interest in lower body musculoskeletal models is well established and is generally based upon a standard order of operations³ (or “pipeline”; Figure 1). Firstly, the inter-segmental forces and moments are calculated by an inverse dynamics analysis based upon a free body diagram of the segments that incorporates the external forces, the linear accelerations of the segments and the rotational kinematics. Secondly, a description of the musculoskeletal geometry of the model is added, which provides detail as to the line of action and moment arms of the muscles. It is then common to assume that the muscles are the sole contributors to the inter-segmental moments, thus an indeterminate problem can be formulated that describes the contribution of each muscle element to the inter-segmental moments previously calculated. In addition, in some models additional constraints are included to further specify the kinematics or kinetics of a given joint. For instance, at the shoulder joint the scapula can be constrained to be in contact with the rib cage or the resultant glenohumeral joint contact force can be compelled to point within the perimeter defined by the labrum to prevent dislocation of the joint (see, for example⁶). This indeterminate problem can then be solved using optimization techniques to give individual muscle forces. The optimization problem is often formulated based upon a consideration of muscular loading alone, and thus neglects a consideration as to the loading of other structures.

Next the individual muscle forces are incorporated within the segmental free body diagram in order to calculate the internal joint contact forces. Finally, it is frequently assumed that the ligaments provide the restraint to the calculated internal joint shear, thus the ligamentous force can be calculated by consideration of the internal forces acting in the free body diagram.

The standard approach to the inter-segmental inverse dynamics analysis is to employ the Newton-Euler iterative method^{40,44}. Firstly, inter-segmental forces are calculated in the laboratory fixed global coordinate frame (GCS). Secondly, the Euler equations of rotational motion are used to calculate the inter-segmental moments in the body fixed segmental coordinate frame (LCS). Traditional inverse dynamics therefore entails multiple coordinate transformations which increase the computational demands of the method. Dumas and colleagues¹⁸ have recently described an alternative approach to the inverse dynamics analysis based upon the use of unit quaternions and wrench notation which allows the complete analysis to be performed in the GCS, thereby greatly reducing the computational complexity of the problem. Recent research has demonstrated that the method of Dumas and colleagues is computationally equivalent to the traditional approach⁸ and thus this alternative formulation presents new opportunities for the analysis of movement. For instance, Cleather and colleagues^{11,10} have recently demonstrated that the equations of motion describing moment equilibrium given by Dumas and colleagues can be modified to include the muscle forces in a musculoskeletal model of the lower limb. This results in an indeterminate problem that can be solved using optimization techniques and gives a more accurate representation as to the function of the biarticular muscles.

The step-by-step approach described in Figure 1 does not recognise the interaction between muscles and ligaments in establishing moment equilibrium. In the first place, at the optimization stage it is assumed that the ligaments do not contribute to the inter-segmental moments observed at the joints. Then, later in the pipeline, indicative ligament forces are calculated based upon the imperative for force equilibrium again based upon the assumption that these forces do not alter the previously established moment equilibrium. This assumption may be valid for some ligaments, for instance, the cruciate ligaments could be reasonably considered to pass through the centre of rotation of the knee however, other ligaments are well placed to contribute to the moment equilibrium of the lower limb. Thus a major limitation of the traditional approach to inverse dynamics is that it precludes an exploration of musculoligamentous interaction.

The pipeline described in Figure 1 has evolved to simplify the computational complexities in determining muscle, ligament and joint reaction forces in the indeterminate musculoskeletal system. This is achieved by partially uncoupling the function of muscles and ligaments to allow the sequential determination of first muscle, then joint reaction and finally ligament forces. In reality, the musculoskeletal system is an integrated whole, and a more realistic representation could be achieved by formulating equations of motion that fully recognize the potential for musculoligamentous interaction, and by solving these equations simultaneously²⁴. To this end, the method of Dumas and colleagues¹⁸ has great utility for posing the equations of motion in a computationally friendly manner. The purpose of this study was therefore to demonstrate the feasibility of using existing technology to simultaneously solve the equations of motion during vertical jumping using

optimization techniques. It was postulated that this approach would permit a more physiologically realistic representation as to the function of the musculoskeletal system than can be achieved by the sequential process represented in Figure 1. A secondary aim was to explore the sensitivity of the solution to the choice of cost function, and in particular the effect of employing cost functions based upon considerations of different tissue loadings (muscular, ligamentous or bony).

Materials and Methods

In this study a previously described^{9,11,10} musculoskeletal model of the right lower limb was employed to explore the use of a new approach to the determination of muscle, ligament and joint contact forces by optimization techniques. The model consists of a 3D description of four linked rigid segments representing the foot, calf, thigh and pelvis. The data analysed in this study pertains to vertical jumps performed by twelve athletic male subjects (mean age 27.1 ± 4.3 years; mean mass 83.7 ± 9.9 kg). The data set was acquired using a Vicon motion capture system (Vicon MX System, Vicon Motion Systems Ltd, Oxford, UK) synchronised with a Kistler force plate (Kistler Type 9286AA, Kistler Instrumente AG, Winterthur, Switzerland). The raw data was captured at 200 Hz and consisted of the position of reflective markers placed on key anatomical landmarks^{36,37} and the ground reaction force. The raw data was filtered using a fifth order Woltring⁴¹ filter prior to analysis. The model is specified by the translations and rotations that describe the position and orientation of each segment, which are calculated using the method of Horn²¹.

The musculoskeletal geometry used in the model is taken from the data set of Klein Horsman and colleagues²² whereas the anthropometry is taken from de Leva¹⁷. The

patella position and orientation is defined to be a function of the knee flexion angle where the data of Klein Horsman et al. is used to find the location of the patellar origin and the work of Nha et al.²⁵ to specify the patellar rotation. The model of Klein Horsman et al. provides a series of via points for each muscle element or ligament that specify its path. These via points include the origin and insertion of the element, but are also used to model the wrapping of muscle over bone by compelling certain muscle elements to pass through one or more fixed points. This anatomical information is used to find the line of action and moment arm of the muscle or ligament, where the line of action is considered to be the vector from the effective insertion to the effective origin, and the moment arm the vector from the centre of rotation to the effective insertion. The effective origin and insertion are defined in Figure 2. Table 1 presents the particular ligaments modelled in this study that were taken from the work of Klein Horsman et al., and details as to the upper bounds of the ligaments that were assumed. Further details as to the description of musculoskeletal geometry in the lower limb model are described in more detail elsewhere^{9,11}.

The method of Dumas and colleagues¹⁸ is used to formulate the equations of motion of the lower limb model. Firstly, the inter-segmental forces can be calculated based on the traditional Newtonian iterative approach (Figure 3a):

$$\hat{S}_i = m_i(\hat{a}_i - \hat{g}) + \hat{S}_{i-1} \quad (1)$$

Next, considering the internal forces acting on each segment (Figure 3b) yields:

$$\sum_{j=1}^U F_j \cdot \hat{p}_{ji} - \sum_{j=1}^U F_j \cdot \hat{p}_{j(i-1)} + \sum_{j=1}^V L_j \cdot \hat{q}_{ji} - \sum_{j=1}^V L_j \cdot \hat{q}_{j(i-1)} + \hat{J}_i - \hat{J}_{i-1} = m_i(\hat{a}_i - \hat{g}) \quad (2)$$

Where $\hat{J}_0 = \hat{S}_0$

The joint reaction forces are considered to act through the joint centres of rotation (which are defined to be located at the intersections of the linked rigid segments). Similarly, the rotational moments are taken around the joint centres of rotation. A consideration of the rotational movement at each segment (Figure 3c) gives:

$$\begin{aligned} \sum_{j=1}^U F_j \cdot \hat{r}_{ji} \times \hat{p}_{ji} - \sum_{j=1}^U F_j \cdot \hat{r}_{j(i-1)} \times \hat{p}_{j(i-1)} + \sum_{j=1}^U b_{ji} F_j \cdot \hat{d}_i \times \hat{o}_{ji} + \sum_{j=1}^V L_j \cdot \hat{s}_{ji} \times \hat{q}_{ji} - \sum_{j=1}^V L_j \cdot \hat{s}_{j(i-1)} \times \hat{q}_{j(i-1)} \\ = m_i \hat{c}_i \times (\hat{a}_i - \hat{g}) + \ddot{\hat{\theta}}_i + \dot{\hat{\theta}}_i \times I_i \dot{\hat{\theta}}_i + \hat{d}_i \times \hat{S}_{i-1} + \hat{M}_{i-1} \end{aligned} \quad (3)$$

$b_{ji} = 1$ for biarticular muscles that cross but do not attach to segment i ;

$b_{ji} = 0$ for all other muscles

$\hat{M}_i = 0$ for $i > 0$

Equation 1 is determinate and thus can be solved directly whereas the remaining equations of motion (Equations 2 and 3) are indeterminate. In this application, the musculoskeletal geometry taken from the Klein Horsman et al.²² data set comprises 163 different muscle elements and 14 ligaments. In combination with the articular contact forces this results in 186 unknown variables. Applying Equations 2 and 3 to the foot, calf and thigh segments in 3D gives a system of 18 equations. This system is solved with an optimization approach using the optimization toolbox of Matlab (version 7.5; The Mathworks, Inc, 2007) employing a cost function adapted from the work of Raikova²⁸:

$$\min_{F_j, L_j, J_j} f = k_1 \sum_{j=1}^U \left(\frac{F_j}{F_{\max_j}} \right)^{n_1} + k_2 \sum_{j=1}^V \left(\frac{L_j}{L_{\max_j}} \right)^{n_2} + \sum_{i=1}^W \left(\sum_{j=1}^3 (k_3 J_j)^{n_3} \right)_i \quad (4)$$

The first term in Equation 4 is based on the cost function of Crowninshield and Brand¹² which is thought to maximise muscular endurance. It is equivalent to minimizing muscle stress, as maximum muscle force is the product of the physiological cross sectional area ($PCSA_j$; taken from the Klein Horsman et al.²² data set) and the maximum muscle stress ($MS=3.139 \times 10^5 \text{ N/m}^2$; taken from Yamguchi⁴³ and doubled to represent the fact that the subjects are from a young athletic population).

$$\min_{F_j} f = \sum_{j=1}^U \left(\frac{F_j}{F_{\max_j}} \right)^{n_1} = \sum_{j=1}^U \left(\frac{F_j}{PCSA_j \times MS_{\max}} \right)^{n_1} = \frac{1}{MS_{\max}^{n_1}} \sum_{j=1}^U \left(\frac{F_j}{PCSA_j} \right)^{n_1} \quad (5)$$

The advantage to calculating muscle forces by using a cost function based upon maximal muscle force is that it provides a simple and consistent way in which to include the ligaments in the optimization (the second term in Equation 4), as the failure strength of the ligaments is known from the literature^{26,30,34} and represents the force upper bound for the ligaments, which could therefore be considered to be comparable to the force upper bound for the muscles. Thus the optimization seeks a solution which both minimizes muscle stress, while seeking to minimize the force in the ligaments relative to their failure limits. If the maximum muscle force is also assumed to be the failure limit of a muscle, the optimization can be characterized as seeking a solution which minimizes the likelihood of any of the tissues approaching

their failure limits, and that there is no bias towards ligamentous or muscular failure. Finally the third term in the cost function represents an imperative to also minimize the joint reaction force²⁸.

The coefficients k_1 , k_2 and k_3 and the exponents n_1 , n_2 and n_3 are chosen to alter the characteristics of the cost function. In this study, five different cases were considered and are presented in Table 1, along with their nomenclature. In particular, by setting the value of the coefficients k_1 , k_2 and k_3 to either 0 or 1, the cost function can be altered by removing selected terms. In this way, the four different cases can be derived. Case 1, includes only the first term of the cost function, and thus is a solution based upon muscle considerations only and is most analogous to the traditional approach (muscles only – MO). Similarly, Case 2 only includes the second term of the cost function, and is thus based solely upon ligament considerations and represents a movement strategy that is based upon the preferential recruitment of muscles to spare ligament loading (ligaments only – LO). Case 4 only includes the third term, and thus relates only to joint reaction force considerations would therefore be predicted to predict the lowest overall muscle and joint activations (joint reaction force only – JO). Case 3 includes both the first and second terms of the cost function (which are given equal weight) and thus relates to both muscle and ligament considerations and was chosen to represent the competing imperatives of muscular and ligament loading (muscles and ligaments – ML). This concept is discussed in more detail later. Finally, Case 5 includes all terms in the optimization, and thus represents the imperative to minimize muscular, ligamentous and joint loading (ALL). The coefficients k_1 , k_2 and k_3 can also be used to weight the relative importance of muscle, ligament or joint reaction force considerations to each other, and the

appropriate weighting is an important aspect was not explored in this study. The exponents n_1 , n_2 and n_3 are higher than those typically employed in the literature^{19,35} however, it has been suggested that the use of higher powers gives a physiologically more realistic solution^{4,29}, and in particular that the muscular activation to external load relationship is linear at higher powers²⁹.

The ligament and joint contact forces presented in this study are those calculated in the optimization procedure (\hat{L}_j and \hat{J}_i respectively). The moments presented are directly calculated as the sum of the cross products of the individual muscle or ligament moment arms with the particular force derived from the optimization solution. The forces and moments calculated in this study were compared to those presented by Cleather and colleagues^{10,11}. Their analysis is based upon the same data set of 12 vertical jumps, but employed a different approach to the calculation of muscle forces. In particular, Cleather and colleagues present forces and moments for the traditional approach (TRAD) and for an optimization based approach similar to that employed in the present work but comprising only muscle elements (that is no ligaments or contact forces including in the optimization algorithm; BI). The differences in muscle and joint forces and muscle moments were analyzed using a one way ANOVA with post hoc Tukey's HSD tests where alpha was set to 0.05 a priori.

Results

The optimization was successful in producing a solution for all cases. Table 3 presents the muscle forces calculated in the five different cases in comparison to the previous results of Cleather and colleagues^{10,11} (TRAD and BI). The peak total

activation of the muscle model was significantly lower for JO than all other cases ($p < 0.05$), a difference that was commensurate with significantly lower activation of gastrocnemius, hamstrings and glutes. Total activation of the muscle model was of similar magnitude for all other cases. The most notable difference at the muscle level was the fact that in all cases the monoarticular plantar flexors had a significantly greater level of activation than that reported previously. Those cases which included a consideration as to muscle stress in the cost function (MO, ML and ALL) showed a trend towards increased activation of the biarticular musculature, and in particular the activation of gastrocnemius in MO was significantly greater than LO, JO, TRAD and BI.

When taken as a whole the magnitude of the peak joint contact forces were consistently lower than that reported previously^{10,11}, although this difference only reached a significant level for JO (Table 4). Notably, MO produced higher contact forces than TRAD and BI, although this difference was not significant. The highest ligament forces were also found in MO, although significant differences were only found for the LCL, ACL and OPL. Ligament loading was smallest for LO and JO, differences that were significant for LCL, ACL and OPL.

Tables 5 and 6 present the peak non-sagittal plane moments created by the ligaments alone, in each of the five cases. The ligaments principally provided adduction and internal rotation moments at the ankle and knee and some abduction moment at the knee. The ligaments of the hip produced minimal non-sagittal plane rotation moments in all cases. Significant differences were only found for MO.

Discussion

In this study a new approach to the calculation of muscle, ligament and joint reaction forces was presented which is based upon formulating a system of indeterminate equations of motion containing all variables of interest, and then solving this system with optimization techniques. This methodology allows a more complete description of the equations of motion to be formulated, and therefore presents a basis for more searching analyses of human movement.

The inter-segmental moments calculated in this study are in agreement with those previously presented in the literature^{2,23,38}. The previous literature relating to internal joint loadings of the knee during jumping is sparse, and is mainly concerned with jump landings however, recent work produced by Cleather⁷ suggests that the magnitude of the joint contact forces during jumping and landing are similar. Studies of jump landings suggest a tibiofemoral joint loading in the range of approximately $17.0 \times BW^{31,32}$. The magnitude of the tibiofemoral joint loading found in this study is over half as great as in these earlier works. Direct in vivo measurements of joint loadings during activities of daily living in patient populations suggest a tibiofemoral loading of $2.0 - 3.0 \times BW^{13,14,15,39}$, rising to $3.0 - 4.5 \times BW^{16}$ in sporting activities. The range of tibiofemoral loadings suggested in this work therefore seems more likely, and the authors would contend that the higher values found in the earlier works are due to a lack of detail in early models. The ligament loadings predicted in this study also seem reasonable when compared to their known failure limits. For instance, this study suggests ACL and PCL loadings of around 250 – 420 N and 1,000

– 2,000 N respectively, which compare favourably with failure limits derived from the literature of 2,000 N^{5,26,42} and 4,500 N¹. Equally, in vitro measurements of ACL loading during simulated jump landing suggest a loading of around 400 N, which is close to the musculoskeletal model derived calculations of Pflum and colleagues²⁷.

The most important limitation of the new method when compared to previous approaches is the fact that in the new model the ligaments are modelled as active force actuators which are able to produce, upon demand, any force up to their failure limit, and without reference to their strain. This approach may at first appear to be grossly unphysiological as a ligament is a passive tissue whose force production is a direct consequence of the strain it experiences. Consequently the force in a ligament should be predicated solely by the position of the joint it spans and thus determined from the musculoskeletal geometry of the model prior to the optimization. This approach is difficult in this type of large scale multi-body model however. The stress-strain relationship of ligaments means that even small length changes can produce large forces and therefore ligament models based purely upon strain considerations tend to be highly sensitive to the specific ligament geometries and the determination of segment positions. Practically, in models specified by generic ligament parameters and based upon optical motion capture techniques, it is challenging to determine changes in ligament length with the precision necessary to use them as inputs to the optimization.

Instead an optimization approach may represent an elegant solution to the problem of determining ligament forces. The motor control strategy developed to perform a

given movement will be chosen based on multiple physiological imperatives. Clearly the articular geometry of a joint will be highly influential, however the motion of the joint will also be guided by the action of muscles and the passive restraint of the ligaments. The optimal movement strategy will therefore be one which is optimal both in terms of the necessity for muscular force production, but also in optimizing ligament strains. The optimal movement pattern may result in a cyclic tensile loading of the ligaments to promote tissue health while preventing ligament strains from exceeding a safe limit. It could be argued that the modelling approach taken in this study can be characterized as capturing these imperatives of a motor control strategy.

Of course, a more robust solution would also capture strain considerations within the solution. For instance, it has been shown that the collateral ligaments of the knee are slack at deeper knee flexion angles²⁰, an experimental finding that is also demonstrated in this model as the collateral ligaments shorten with knee flexion angle. However, in contrast with the ligament length data in the present model the collateral ligaments are available as force actuators throughout the full range of knee motion. The effect of this is that the current solutions sometimes predict a collateral ligament recruitment that is not physiological. The remedy to this problem lies in finding methods for incorporating length considerations within the solution. In a preliminary study Cleather⁷ has demonstrated that strain considerations can be used to specify further bounds for the optimization which improve the realism of the muscle model. Alternatively, if ligament forces can be derived by other means (for instance through more accurate length measurements derived from imaging techniques or the use of alternative models), then these forces can be incorporated within the optimization solution. For instance, Southgate³³ has recently incorporated ligament

1 forces derived from a ligament model within an optimization procedure in order to
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3 evaluate muscular forces at the shoulder joint. This discussion should not detract
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5 from the principal argument of this paper, that optimization solutions to the inverse
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7 dynamic problem should incorporate all equations of motion, rather that future
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9 research should seek to populate the optimization problem with further detail (in the
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11 form of further constraints or equations) derived from kinematic and kinetic
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13 considerations.
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20 A pivotal question when employing this methodology is the appropriate choice of cost
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22 function for the ligaments. The cost function should both have a physiological
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24 meaning in representing the manner in which tension is likely to be controlled in the
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26 ligaments in isolation, but also in the representing the interaction between muscular
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28 and ligamentous forces. This work highlights various interesting questions. Firstly,
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30 what criteria are used to regulate the force experienced by the ligaments and how can
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32 these be best described by a cost function, given the assumption that motor control
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34 strategies are chosen to minimize the force in the ligaments? Secondly, what is the
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36 interaction between muscular and ligamentous forces during movement, and in
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38 particular, do muscle or ligament force considerations dominate motor control
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40 strategies? How can the relative weight of muscle or ligament considerations be
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42 reflected in a cost function, given an optimization approach to deriving force
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44 estimates?
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55 A further limitation of the present work is the lack of dynamic muscle modelling; that
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57 is, that the force generating capacity of the muscles is assumed to be constant and
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independent of the known force-length and force-velocity relationships. This limitation may be particularly pertinent in high acceleration activities like vertical jumping where the time taken to develop force may be an important determinant as to the movement strategy employed. Equally a consideration of muscle dynamics is important when considering the stability and musculoligamentous interaction at the knee. Whereas muscles will require time to develop force in response to perturbations, the ligaments are immediately responsive to position changes due to the stress-strain relationship. Future work should therefore seek to incorporate muscle dynamics within the optimization solution of this type of model.

It is important to recognise that in the previous work of Cleather and colleagues¹¹, in order for the optimization to produce a solution it was necessary to increase the upper bounds for 6 of the 12 subjects, and for some subjects the necessary upper bound was over double that used in this study. The increased upper bounds were utilized solely by the small muscle groups and represented the difficulty in finding an equilibrium solution in 3D (and in particular in the non-sagittal planes). This is a consequence of the fact that the solutions of this type of model are highly sensitive to the number and variability of force actuators available to the optimization algorithm⁹. In particular, musculoskeletal models of the lower limb tend to have a limited number of muscle elements that can provide non-sagittal plane rotations at the knee and it is common for these types of models to constrain the knee to 1 DOF. The availability of the five major ligaments of the knee (as well as ligaments of ankle and hip) as force actuators within this model markedly increases the probability of finding an optimal solution (and equally that this solution will have a lower overall level of activation). In this study, the ability of the ligaments to provide rotational moments, ameliorated this

problem, and it was possible to find a solution with upper bounds derived solely from a consideration of physiological cross-sectional area. The physiological implication of this finding is to emphasize the importance of the ligaments in creating joint stability and may suggest that the maintenance of joint stability by muscular activity alone, particularly at the knee, is challenging due to an insufficient number of muscular moment arms with positions and orientations that can provide the required stabilizing moments. Additionally it is important to recognize the role of the ligaments in providing moments that are less optimally produced by muscle function alone (which is a product of their position and orientation). The stability of the knee is thus dependent on the synergy between muscles and ligaments and in particular their relative musculoskeletal geometry.

The results of this study suggest that the ligaments have a clearly delineated role in creating stability at the ankle and knee during vertical jumping. Specifically, the ligaments appear to be most consistently recruited in providing adduction and internal rotation moments at these joints. This type of understanding as to the different roles of the muscles and ligaments in creating joint stability can only be ascertained when both structures are modelled as simultaneously contributing to force and moment equilibrium, as demonstrated in this work.

The joint contact forces predicted in JO are lower than those found in all other cases. This is unsurprising in as much as the cost function in JO is solely based upon an imperative to minimize joint contact forces. This is achieved by a commensurate reduction in muscular cocontraction (when compared to the other cases) as indicated

by the magnitude of the peak total muscle force. Equally, LO and JO relied less heavily upon the recruitment of the ankle and knee ligaments to provide non-sagittal plane rotation moments, which is again a consequence of the cost function based imperative to reduce ligament or joint contact force loading. In contrast, MO produced greater total activation, despite a muscle focussed cost function. This somewhat counter-intuitive result is due to the fact that a cost function that minimizes muscle stress can increase muscular activation due to the strong imperative for force sharing among all involved musculature. These results demonstrate the key sensitivity of muscle, ligament and joint contact forces to the choice of cost function – and consequently the physiological imperatives that determine motor control strategies. The use of a cost function similar to the one employed in this study allows the exploration of these effects.

The new technique was capable of producing a viable solution for a variety of different cost functions. The technique therefore has utility for improving the understanding of human movement. In particular, the results of this study demonstrate that an approach that captures the musculoligamentous interaction involved in creating joint stability may result in solutions involving more realistic maximum muscle forces and the calculation of joint contact forces that are lower than have previously been reported. More importantly, this research demonstrates that current technology permits the simultaneous solution of the equations of motion of human activity. Future research should therefore seek to pose inverse dynamics models in terms of the complete, simultaneous equations of motion in order to fully explore the nature of joint stability.

Conflict of Interest Statement

There are no conflicts of interest.

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Definition of terms

\hat{a}_i	linear acceleration of the centre of mass
\hat{c}_i	vector from the proximal joint to the segment COM
\hat{d}_i	vector from the proximal to the distal joint
f	cost function
F_j	magnitude of force in muscle
\hat{g}	acceleration due to gravity
i	segment/joint number (numbering from distal to proximal)
\hat{I}_i	inertia tensor
j	muscle or ligament number
\hat{J}_i	joint contact force at proximal end of segment
k_1, k_2, k_3	cost function coefficients
L_j	magnitude of force in ligament
\hat{m}_i	mass of segment
\hat{M}_i	inter-segmental moment at proximal end of segment
n_1, n_2, n_3	cost function exponents
\hat{o}_{ij}	line of action of biarticular muscle j about segment i
\hat{p}_{ij}	line of action of muscle j about joint i
\hat{q}_{ij}	line of action of ligament j about joint i
\hat{r}_{ij}	moment arm of muscle j about joint i
\hat{S}_i	inter-segmental force at proximal end of segment
\hat{s}_{ij}	moment arm of ligament j about joint i

An optimization based simultaneous approach

U	total number of muscles
V	total number of ligaments
W	total number of joints
$\dot{\theta}_i$	angular velocity of segment
$\ddot{\theta}_i$	angular acceleration of segment

Table 1. Ligaments included in the described model.

Ligament	Joint	Upper bound (N)
Iliofemoral ligament (anterior)	Hip	850
Iliofemoral ligament (lateral)	Hip	850
Pubofemoral ligament	Hip	450
Ischiofemoral ligament	Hip	450
Anterior cruciate ligament (ACL)	Knee	3000
Posterior cruciate ligament (PCL)	Knee	2000
Medial collateral ligament (MCL)	Knee	2000
Lateral collateral ligament (LCL)	Knee	4000
Oblique popliteal ligament (OPL)	Knee	1000
Posterior tibiotalar ligament	Ankle	850
Tibiocalcaneal ligament	Ankle	850
Tibionavicular ligament	Ankle	850
Posterior talofibular ligament	Ankle	850
Calcaneofibular ligament	Ankle	850

Table 2. Parameters used within the optimization cost function for the four different cases considered in this study (Cases 1-5), and for the previous work of Cleather and colleagues¹¹.

Case	k_1	k_2	k_3	n_1	n_2	n_3
1. Muscles Only (MO)	1	0	0	30	-	-
2. Ligaments Only (LO)	0	1	0	-	30	-
3. Muscles + Ligaments (ML)	1	1	0	30	30	-
4. JRF Only (JO)	0	0	1	-	-	2
5. ALL	1	1	10^{-4}	30	30	30
6. TRAD	1	0	0	30	-	-
7. BI	1	0	0	30	-	-

Note: although the TRAD and BI cases use comparable cost functions, the optimization problem is formulated differently (see Cleather et al.¹¹ for more details).

Table 3. Peak muscle forces (\times BW) calculated during vertical jumping for selected muscles ($^n = p < 0.05$, when compared to Case n; $* = p < 0.05$, for the comparison with all cases).

	Biarticular			Monoarticular			Total	Total (+ Ligaments)
	Gastroc	Rec Fem	Hamst	Sol + Tib P	Vastus	Glutes		
1. Muscles Only	$2.0 \pm 1.3^{2,4,6,7}$	0.5 ± 0.4	1.1 ± 0.3^4	$6.5 \pm 1.3^{6,7}$	2.6 ± 0.4	3.0 ± 1.1^4	16.1 ± 3.3^4	19.5 ± 4.5^4
2. Ligaments Only	1.0 ± 0.5^1	0.4 ± 0.2	1.0 ± 0.3^4	$6.6 \pm 1.2^{6,7}$	2.5 ± 0.4	2.4 ± 0.8	15.1 ± 2.9^4	17.0 ± 3.7^4
3. Muscles + Ligaments	1.2 ± 0.6^4	0.5 ± 0.2	1.0 ± 0.3^4	$6.6 \pm 1.2^{6,7}$	2.6 ± 0.5	2.6 ± 0.8^4	15.6 ± 2.6^4	18.7 ± 4.3^4
4. JRF Only	$0.4 \pm 0.3^{1,5}$	0.4 ± 0.3	$0.5 \pm 0.3^{1,2,3,5}$	$6.2 \pm 1.2^{6,7}$	2.2 ± 0.4	$1.4 \pm 0.4^{1,3,5}$	$11.0 \pm 1.6^*$	$11.9 \pm 2.0^*$
5. ALL	1.3 ± 0.8^4	0.5 ± 0.3	1.0 ± 0.3^4	$6.6 \pm 1.2^{6,7}$	2.5 ± 0.4	2.6 ± 0.8^4	15.3 ± 2.3^4	18.2 ± 3.6^4
6. TRAD	0.7 ± 0.3^1	0.2 ± 0.1	0.7 ± 0.3	$4.4 \pm 1.0^{1,2,3,4,5}$	2.3 ± 0.4	2.3 ± 0.8	15.6 ± 2.7^4	-
7. BI	0.7 ± 0.4^1	0.4 ± 0.4	0.8 ± 0.5	$4.4 \pm 1.1^{1,2,3,4,5}$	2.2 ± 0.4	2.4 ± 1.1	15.8 ± 3.8^4	-

Note: total represents the peak forces for all force actuators combined (for muscles alone and for muscles + ligaments).

Table 4. Peak joint contact forces and ligament forces (\times BW) calculated during vertical jumping ($^n = p < 0.05$, when compared to Case n; $* = p < 0.05$, for the comparison with all cases).

Case	AF	TFJ	HF	MCL	LCL	ACL	PCL	OPL
1. Muscles Only	9.1 \pm 2.5	8.5 \pm 1.9 ⁴	6.9 \pm 2.2 ⁴	0.4 \pm 0.2	1.6 \pm 0.7 ⁴	0.5 \pm 0.2 ⁴	2.2 \pm 1.5	0.7 \pm 0.4 ^{2,4}
2. Ligaments Only	8.7 \pm 1.6	6.8 \pm 1.7 ⁴	5.4 \pm 1.0 ⁴	0.3 \pm 0.1	1.0 \pm 0.6	0.3 \pm 0.2 ⁴	1.1 \pm 0.8	0.2 \pm 0.3 ^{1,3}
3. Muscles + Ligaments	9.0 \pm 1.6	7.4 \pm 2.1 ⁴	5.5 \pm 1.0 ⁴	0.4 \pm 0.2	1.2 \pm 0.8	0.5 \pm 0.4 ⁴	1.7 \pm 0.9	0.7 \pm 0.5 ²
4. JRF Only	7.5 \pm 1.5	4.2 \pm 0.6*	3.4 \pm 0.8*	0.3 \pm 0.2	0.7 \pm 0.4 ¹	0.0 \pm 0.1*	1.3 \pm 0.9	0.2 \pm 0.4 ¹
5. ALL	9.0 \pm 1.5	7.4 \pm 2.0 ⁴	5.5 \pm 0.9 ⁴	0.3 \pm 0.3	1.3 \pm 0.8	0.6 \pm 0.4 ⁴	1.7 \pm 0.9	0.6 \pm 0.5
6. TRAD	9.1 \pm 1.8	8.1 \pm 2.0 ⁴	5.8 \pm 1.3 ⁴					
7. BI	9.0 \pm 1.8	8.0 \pm 1.9 ⁴	6.0 \pm 1.3 ⁴					

Table 5. Peak internal/external rotation moments (\times BW) created by the ligaments during vertical jumping ($^n = p < 0.05$, when compared to Case n).

Case	Ankle		Knee		Hip	
	Internal	External	Internal	External	Internal	External
1. Muscles Only	0.20 \pm 0.09	0.07 \pm 0.04	0.60 \pm 0.27 ⁴	0.07 \pm 0.05	0.09 \pm 0.05*	0.01 \pm 0.01 ⁴
2. Ligaments Only	0.20 \pm 0.11	0.05 \pm 0.03	0.38 \pm 0.21	0.05 \pm 0.05	0.02 \pm 0.01 ¹	0.00 \pm 0.00
3. Muscles + Ligaments	0.23 \pm 0.12	0.06 \pm 0.03	0.43 \pm 0.26	0.06 \pm 0.04	0.04 \pm 0.03 ¹	0.01 \pm 0.00
4. JRF Only	0.16 \pm 0.10 ¹	0.04 \pm 0.04	0.29 \pm 0.15 ¹	0.04 \pm 0.03	0.01 \pm 0.01 ¹	0.00 \pm 0.00 ¹
5. ALL	0.23 \pm 0.12	0.06 \pm 0.03	0.43 \pm 0.26	0.06 \pm 0.04	0.04 \pm 0.02 ¹	0.00 \pm 0.00

Table 6. Peak ad/abduction moments (\times BW) created by the ligaments during vertical jumping ($^n = p < 0.05$, when compared to Case n).

Case	Ankle		Knee		Hip	
	Add	Abd	Add	Abd	Add	Abd
1. Muscles Only	0.15 \pm 0.07	0.09 \pm 0.10	0.25 \pm 0.12	0.20 \pm 0.11	0.08 \pm 0.08	0.09 \pm 0.13*
2. Ligaments Only	0.12 \pm 0.08	0.05 \pm 0.03	0.16 \pm 0.10	0.13 \pm 0.08	0.03 \pm 0.02	0.01 \pm 0.01 ¹
3. Muscles + Ligaments	0.14 \pm 0.07	0.09 \pm 0.08	0.21 \pm 0.12	0.15 \pm 0.11	0.06 \pm 0.09	0.03 \pm 0.03 ¹
4. JRF Only	0.10 \pm 0.08	0.02 \pm 0.02	0.17 \pm 0.12	0.14 \pm 0.08	0.02 \pm 0.03	0.00 \pm 0.00 ¹
5. ALL	0.14 \pm 0.07	0.09 \pm 0.08	0.21 \pm 0.12	0.15 \pm 0.10	0.05 \pm 0.04	0.02 \pm 0.02 ¹

Figure Legends

Figure 1. Traditional approach to the calculation of muscle, ligament and joint contact forces.

Figure 2. Diagram depicting the effective origin and insertion of a muscle element. The filled rhomboids represent muscle via points, whereas the dotted boxes enclose the via points that are considered to be located in a given segment. The effective origin is the most distal via point in the proximal segment and the effective insertion is the most proximal via point in the distal segment.

Figure 3. Nomenclature used to describe segmental kinematics and kinetics.

Table Legends

Table 1. Ligaments included in the described model.

Table 2. Parameters used within the optimization cost function for the four different cases considered in this study (Cases 1-5), and for the previous work of Cleather and colleagues¹¹ (Cases 6 and 7).

Table 3. Peak muscle forces (\times BW) calculated during vertical jumping for selected muscles ($^n = p < 0.05$, when compared to Case n; $^* = p < 0.05$, for the comparison with all cases).

Table 4. Peak joint contact forces and ligament forces (\times BW) calculated during vertical jumping ($^n = p < 0.05$, when compared to Case n; $^* = p < 0.05$, for the comparison with all cases).

Table 5. Peak internal/external rotation moments (\times BW) created by the ligaments during vertical jumping ($^n = p < 0.05$, when compared to Case n).

Table 6. Peak ad/abduction moments (\times BW) created by the ligaments during vertical jumping ($^n = p < 0.05$, when compared to Case n).

Figure 1. Traditional approach to the calculation of muscle, ligament and joint contact forces.

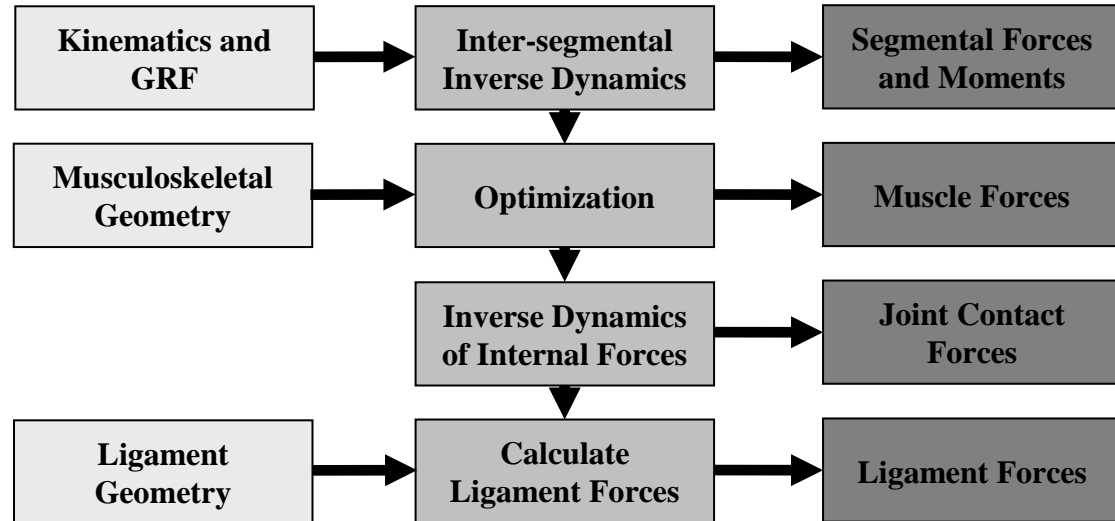


Figure 2. Diagram depicting the effective origin and insertion of a muscle element. The filled rhomboids represent muscle via points, whereas the dotted boxes enclose the via points that are considered to be located in a given segment. The effective origin is the most distal via point in the proximal segment and the effective insertion is the most proximal via point in the distal segment.

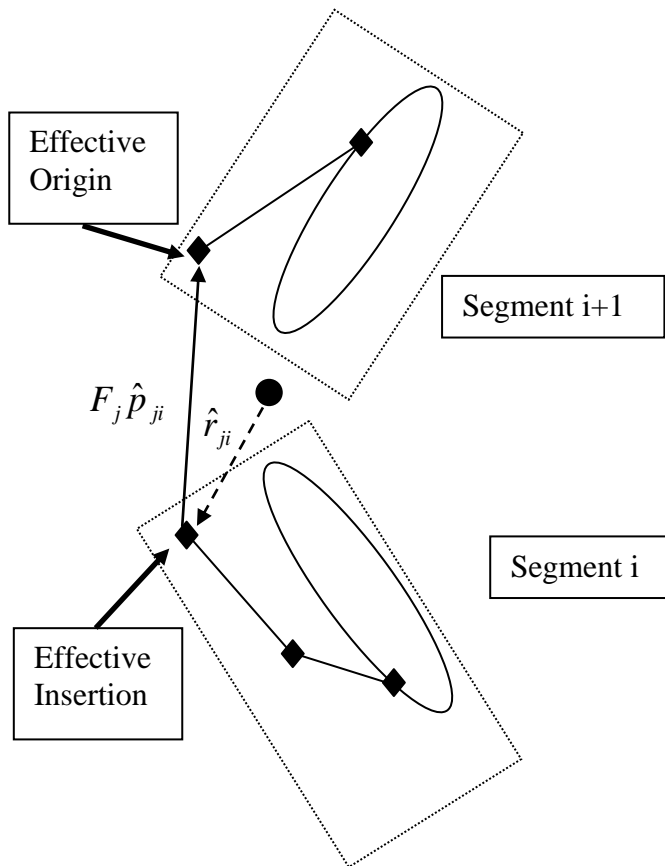


Figure 3. Nomenclature used to describe segmental kinematics and kinetics in Equations 1-3.

Figure 3a. Nomenclature related to Equation 1 (external force equilibrium).

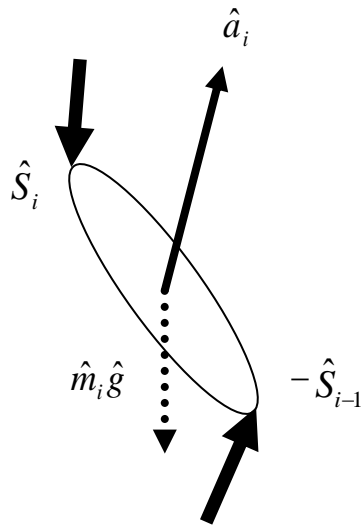
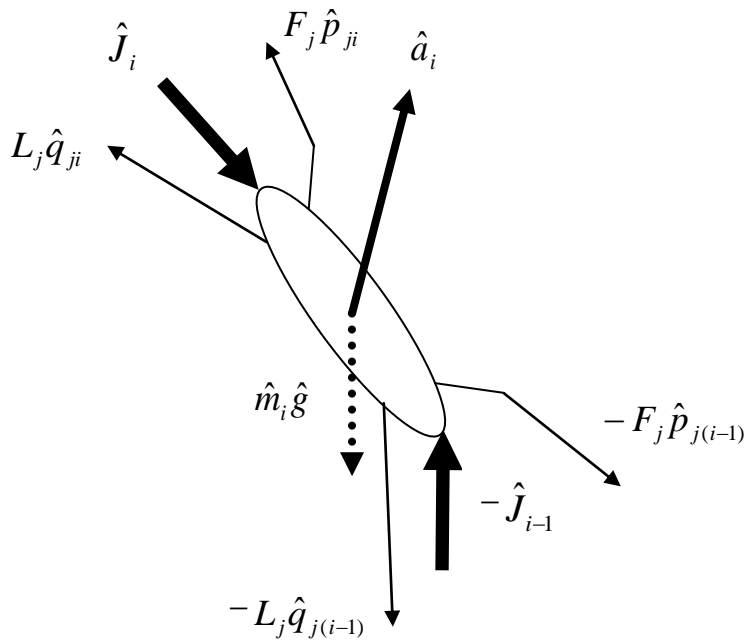
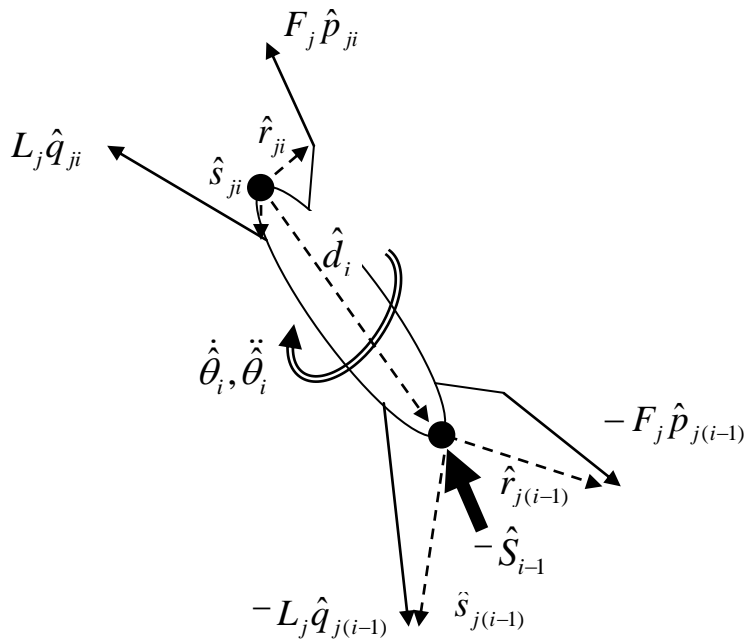


Figure 3b. Nomenclature related to Equation 2 (internal force equilibrium).



Note: the change in direction of the muscle forces represents the wrapping of muscle around bone and soft tissue and is modelled by the use of via points.

Figure 3c. Nomenclature related to Equation 3 (internal moment equilibrium).



Note: the change in direction of the muscle forces represents the wrapping of muscle around bone and soft tissue and is modelled by the use of via points.