Understanding the relations between Transcendence and Mathematics: A Resource Essay for educators and students in Catholic universities to appreciate its deep meanings

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Mathematics was considered to be a universal language that God used to write the book of nature. Many of the greatest mathematicians (such as Descartes, Leibniz, Euler, Cantor) saw their mathematical work as a way to have a clearer insight into the existence of God and His infinity, as well as to glorify His name. This paper explores the mathematics-theology relation in the works of some of the greatest mathematicians from the 15th century to the present day. It also discusses how this information could be used to introduce the investigation of the reality of mathematics as divine language in the mathematics curricula of Catholic universities and colleges. At advanced levels, students need to understand Mathematics not only as a secular subject of technical utility, but also as a rich culture in which ideas of transcendence can be explored.

Keywords: mathematics; physics; theology; philosophy; infinity; education; transcendence

Introduction

Mathematics. What a scary word! As soon as people hear it, they "shrink away" because they associate it with some sort of unthinkable abstraction, complex equations, and difficult calculations that only "brainy" people can tackle. This is mainly because through the centuries the place of mathematics in human knowledge has dramatically changed from playing a fundamental role together with natural sciences, philosophy and theology in the understanding of the universe and its laws, to being largely separated from any other discipline. As a consequence, contemporary education does not cover the pivotal role that mathematics can play in understanding questions deeper than 'how to get a good job' or progress in corporate finance.

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The Catholic position is that a full understanding of the cosmos can be achieved only by means of the exploration of its Divine Creator. Numbers are the letters composing the divine alphabet used by God to speak a sensorial language that may be understood by our limited minds. Through numbers and mathematical expressions God is able to show us the path to the full understanding of the cosmic truths and of the ultimate meaning of human existence.

In this paper, I will explore how the mathematics-theology relation has evolved from the 12th century to the present day, and what insight into the deeper meaning of mathematical ideas/theories, the transcendent nature of mathematics had. History and philosophy of mathematics show us the pivotal role played by the Christian faith in profound mathematical inquiries. The examples of great mathematicians who saw the 'hand of God' in their works should encourage Catholic universities and colleges to rethink their mathematics curricula so that they contain some understanding of the relation between transcendence and mathematics.

Different types of mathematics

In ancient times, the sacred and transcendent nature of numbers and their mathematical relations were considered of uttermost importance in answering "What is God?". For example, in his book *De consideratione* (On Consideration) published in 1150, the French Christian theologian St Bernard de Clairvaux (1090 – 1153) tells us that God is *"length, breadth, height, and depth"*. However, Bernard explains that *"no divisions of the Substance are expressed in that fourfold enumeration"* (St Bernard 2015, Ch. XIII, p. 165) because that quaternary is simply a tool used by our limited mind to conceive God. This *arithmetical* approach to theology was possible because ancient thinkers were considering only numbers and geometric shapes rather than modern mathematics, such as infinitesimal calculus,[1] complex numbers,[2] differential geometry,[3] etc.

In the Middle Ages, two types of mathematics were considered:

- *mathematics proper*

- *secondary mathematics*

Both kinds of mathematics were abstract investigations but they differ on the object of their exploration. *Mathematics proper* inquired about substances, whilst *secondary mathematics* probed the properties of substances. Mathematicians were dealing with secondary mathematics, whereas 'natural philosophers' were engaging with mathematics proper. Hence, secondary mathematics was considered inferior to mathematics proper because, as explained by the philosopher and bishop of Chartres John of Salisbury (1120 – 1180), *"after form has been abstracted from matter [...] it is futile to try to attire matter with [...] properties which it cannot bear, or to divest matter of clothing that it does not possess. Anyone who presumes to exceed this limitation is no longer considering the constitution of nature. He is rather dealing with the figments of a mind that is involved in mathematical subtleties"* (John 2009, p. 159). The view that secondary mathematics is unsuitable to understand nature because it is based on pure logical reasoning with no connection to the world, is reminiscent of the modern concept of mathematics as *"a pure logical creation, 'undefiled' by contact with human emotions or religious feelings"* (Davis 2004).

The net separation between mathematics and theology is a consequence of the Enlightenment movement[4] of the 18th century, when the sciences and mathematics divorced themselves from any religious and faith-related ideas, and more recently of positivistic philosophies[5]. This separation is clearly exposed in the famous answer *"Sire, I do not need [God's] hypothesis"* (Hahn 2005) that allegedly the French mathematician, scientist and philosopher Pierre-Simon Laplace (1749 – 1827) gave to Napoleon Bonaparte when the latter pointed out to Laplace that he could not find God mentioned in any of Laplace's scientific writings.

Mathematics: A universal language

Mathematics is the most abstract and, therefore, the most universal of all the languages because it does not relate to any concept that humans can easily understand or relate to. The Hungarian physicist Eugene Wigner (1902 – 1995) emphasised the universality of mathematics when he explored *"the unreasonable effectiveness of mathematics in the natural sciences"* (Wigner 1960, p.1) and concluded that *"the miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve"* (Wigner 1960, p.14). He also auspicated that in future research the fundamental role of mathematics could not only be preserved in the sciences but could also used in many other disciplines.

Mathematical beauty and the divine design in 20th century mathematics and physics

The mathematical nature of the universe was also acknowledged by the English physicist and mathematician Paul Dirac (1902 – 1984) who famously quoted that *"God used beautiful mathematics in creating the world"* (Pagels 1982, p. 295). Dirac's idea that mathematical beauty was an expression of God's thoughts was shared by the English theoretical physicist and theologian John Polkinghorne (1930 – 2021) who posited that only theories that can be expressed in terms of beautiful mathematical equations have a high degree of certitude and verisimilitude (Polkinghorne 2011, p. 141).

The English physicist and mathematician Arthur Eddington (1882 – 1944) was a devout Christian who believed in the importance of mystical experience. In particular, he posited that the physical world is separated from the spiritual one and this is why we cannot access the latter by means of empirical methods. For Eddington, physics was not equipped to explore the spiritual world a feat that only mystical experience can achieve (Eddington 1928, p. 275). Conversely, the German theoretical physicist Werner Heisenberg (1901 – 1976), who was also a faithful Christian, believed that every scientific discovery gives us a deeper insight into God's plan. He believed that *"one is almost scared by the simplicity and harmony of those connections which nature suddenly spreads out in front of you and for which you were not really prepared [and] [...] when one stumbles these very simple, great connections which are finally fixed into an axiomatic system the whole thing appears in a different light. Then our inner eye is suddenly opened to a connection which has always been there – also without us – and which is quite obviously not created by man"* (Heisenberg 1971, p. 150).

A Theory of Everything and the mathematics-faith dialogue in the 21st century

In contemporary science many physicists and mathematicians dissociate completely mathematics and theology. For example, the Swedish-American physicist Max Tegmark (1967 –) explored the mathematical nature of the universe by comparing two hypotheses about reality:

- External Reality Hypothesis (ERH): *"there exists an external physical reality completely independent of us humans"*;

- Mathematical Universe Hypothesis (MUH): *"our external physical reality is a mathematical structure"* (Tegmark 2008, p. 101).

ERH is the hypothesis usually accepted and subscribed by many but not all physicists. Those who reject it is because they follow the Copenhagen interpretations of quantum physics that reality exists only when observed[6]. The ERH is based on the universality of the mathematical language used to write the laws of nature because it implies that the way the physical world works and behaves can be understood by non-human entities, such as computers. In physics, the more general and all-encompassing the theory, the better, and this is why physicists are still searching for a *Theory of Everything*, namely for one single theory that can explain the works and behaviours of the universe. Such a theory must be highly mathematical because in order to be capable to explain all the natural phenomena it must be as abstract as possible.

A Theory of Everything is what the MUH is implying because it states that our physical world *is* a *mathematical structure*, namely *"abstract entities with relations between them"* (Tegmark 2008, p. 102). Hence, not only the MUH is valid regardless the use of any concept that humans can understand but it also alters our perception of the universe because it shows us that external reality is not simply described by means of numbers and mathematical expressions but it *is* mathematics. Therefore, the ordered and mathematical patterns that we observe in nature are simply a consequence of the fact that nature *is* mathematics and are not to be interpreted as evidence of a divine design of the cosmos.

It is worth noticing that the quest for a Theory of Everything does not necessarily negate the possibility of unravelling the presence of God in Creation. Recently, the American theoretical physicist Michio Kaku (1947 –) in his book *The God Equation* published in 2021, accompanies us in a journey through the historical and scientific labyrinth of the quest for a single, compact, and beautiful mathematical equation that could not only explain everything that happens in the universe but also allows us to *"read the mind of God"* (Kaku 2021, p. 2).

Mathematics and Theology

Throughout the centuries the universality of mathematics has led to its development within a framework permeated of religious ideas, concepts and thoughts. Theologies based on mathematics allow for the description of the abstract laws that govern the cosmos created by God, a concept clearly expressed by St Paul who in his Letter to the Ephesians 3:18 hopes that the Father will grant the Ephesians the *"power to comprehend with all the saints what is the breadth and length and height and depth"* [7].

The first thinkers for whom mathematics played a central role in life and religion were the Pythagoreans[8], who attributed the order and regularity expressed in the cosmos to the existence of numbers. They considered numbers not only as vessels, which contained the eternally unchanging laws of the universe, but also as the intelligible manifestation of those divine and eternal laws. The Pythagoreans considered the first ten numbers as the only numbers manifesting the eternal laws of the universe created by God because all successive numbers are built using the first decade; for example, the number 11 is obtained by adding 10 to 1, or 9 to 2, or 8 to 3, or 7 to 4, or 6 to 5.

The divine nature of the first ten numbers was explored by the Syrian Neoplatonist philosopher Iamblichus (245 – 325) in his book *The Theology of Arithmetic* where he stated that *"number is the form of things"*, and concluded that *"the creative mind wrought the construction and composition of the universe and everything in the universe by reference to the likeness and similarity of number, as if to a perfect paradigm"* (Iamblichus 1988, p. 109).

The Greek philosopher Plato (ca. 429 – ca. 347 BC) described the strong relation between mathematics and the Divine in his scientific treatise *Timaeus*. Plato stated that out of the four elements of fire, air, water, and earth *"the body of the Cosmos was harmonised by proportion and brought into existence [...] so that being united in identity with itself it became indissoluble by any agent other than Him who had bound it together"* (Plato 1929, 32C). Plato posited that the divine craftsman (demiurge)[9] made the cosmos by using arithmetic and geometric sequences of numbers that result in harmonies that tended to represent intelligibly the perfection of the divine pattern on which the physical world was moulded.

St Augustine of Hippo (345 – 430) talked about the sacred nature of mathematics in many of his treatises but his most famous comments and ideas on this subject are to be found in Book II of *De Libero Arbitrio* where he says that *"the intelligible structure and* *truth of numbers does not pertain to the bodily senses. It remains pure and unchangeable"* (Augustine 2010, 2.8.24.93, p. 49). Since numbers do not change, they are related to Wisdom because *"just as there are true and unchangeable rules of numbers [...] so too are there true and unchangeable rules of wisdom"* (Augustine 2010, 2.10.29.119, p. 54). Augustine states that everything in the physical world is built by using mathematical expressions and correlations because eternal mathematical truths come from God who created the universe, a concept clearly expressed in the Book of Jeremiah 33: 25 when God says that He *"established the fixed laws that govern heaven and earth"*.

In the 12th century, the French philosophers Thierry of Chartres (1100 – 1150) followed in the footsteps of Augustine and in his treatise on the interpretation of the first six days of creation, Thierry explained that because every number is created by the repetition of unity, then unity was omnipotent in creating numbers; and because *"the creation of numbers is the creation of things"* (Bernard 2004), then unity was omnipotent in creating things; *ergo*, unity was omnipotent.

The idea that the laws of God's cosmos are governed by numbers and their mathematical connections it is not be considered irrational or unscientific. On the contrary, it is an idea that is based on concrete evidences, such as the famous mathematical relationship between energy and mass – $E = mc^2$ where E is the energy, m is the mass and c is the speed of light in vacuum – developed by the German physicist Albert Einstein (1879 – 1955), or the empirical formula for predicting the distance between a planet and its Sun – known as Bode's law[10] – that was developed by the German astronomer Johann Daniel Titius (1729 – 1796) and later made known to the public by Johann Elert Bode (1747 – 1826). Another example of the mathematical nature of the physical world is the double helix of the DNA structure, which is a mathematical structure composed of two continuous curves (DNA strands) whose interweaving is due to a so-called *linking number*. The mathematical arrangement of a closed DNA structure is due to *"the biochemical nature of the strands [which] guarantees that during closure each strand of the DNA can only bind to itself"* (Swigon 2009, p. 296). As the German philosopher and mathematician Gottfried Wilhelm Leibniz (1646 – 1716) stated in his *Dialogue de la conexión entre les mots et les choses*[11] (A VI, 4, 22) published in 1677, *"cum Deus calculate et cogitationem exercet, fit mundus"* (when God thinks things through and calculates, the world is made).

Leibniz was a strong supporter of the close relation between faith and reason and believed that God endowed human beings with a rational mind to give them the necessary tools to comprehend God and His works. For Leibniz, *"God is all order; he always keeps truth of proportions, he makes universal harmony; all beauty is an effusion of his rays"* (Leibniz 1996, p. 51).

God and infinity: Nicholas of Cusa, Georg Cantor

One of God's attributes is infinity. In the *Summa Theologiae* the Italian theologian Thomas Aquinas (1225 – 1274) says that God is infinite because His being is self-subsisting (Aquinas 1981, 1a.7.1). However, there is no a unique definition of infinity.

Nicholas of Cusa

The German philosopher, theologian and mathematician Nicholas of Cusa (1401 – 1464) placed great emphasis on the mathematical and geometric concept of infinite within his theology. He defined finite things as things they are what they are and nothing else (for example, an apple is an apple and it is not, say, a pear or a strawberry). Conversely, Nicholas defined God as *Not-Other* because God is not different from other, but He is other (for example, God is not other than an apple but He is the apple in the same way as He is a pear, He is a strawberry, etc). So *Not-Other* is the *name* that Nicholas gave to God (Miller 2015, p. 25). Since *other* refers to finite objects that belong to the finite/physical world, then *Not-Other* identifies with *infinite* because it is the negation of other, namely of finite (Celeyrette 2011, p.157).

Mathematics played a fundamental role in Nicholas' thought because the certainty and reliability that come from mathematics are pivotal for gaining a certain knowledge of things. Such a high level of mathematical certainty is possible because mathematics allows the mind to work with abstract concepts such as numbers, and so it does not rely on anything as uncertain and changeable as physical reality (Murawski 2016, p. 100). Abstract realities, such as numbers and geometric shapes, are only in the mind of God because God *"arranged all things by measure and number and weight"* (Wis. 11: 20). In his treatise *De Docta Ignorantia* (On Learned Ignorance) published in 1440, Nicholas says that *"in creating the world, God used arithmetic, geometry [...] For through arithmetic God united things. Through geometry He shaped them, in order that they* *would thereby attain firmness, stability, and mobility in accordance with their conditions"* (Cusa 2007, II.13, p. 119). Nicholas saw mathematics as the bridge between physics, which deals with what is material, finite, and uncertain, and theology which considers realities that are not physical. Although mathematical objects are not completely fixed and they can undergo some changes, they can still be considered certain because they belong to the realm of the mind and objects that *"are more abstract than perceptible things, viz., mathematical […] are very fixed and are very certain to us"* (Cusa 2007, I.11, p. 25).

For Nicholas, mathematics can help us to understand God' infinity by means of mathematical symbols. In particular, Nicholas uses the relationship between opposites (for example, finite line and infinite line) to explain how mathematical knowledge can bring us closer to God's knowledge (Murawski 2016, p. 106). For example, if we consider a finite line, then *"every finite line has its being from the infinite line, which is all that which the finite line is"* (Cusa 2007, II.5, p. 75).

Georg Cantor

The German mathematician Georg Cantor (1845 – 1918) is famous for formulating the transfinite set theory[12], which revolutionised the mathematical landscape by allowing us to understand the nature of infinity and how to treat it mathematically. Cantor encountered fierce opposition from the mathematical community, with some of the greatest mathematicians of the time attacking him and his transfinite numbers. For example, the French mathematician and philosopher Henri Poincaré (1854 – 1912) saw Cantor's transfinite set theory as a *"'a disease' from which mathematics would someday be cured"*, and the German mathematician Leopold Kronecher (1823 – 1891) *"attacked Cantor personally, calling him a 'scientific charlatan', a 'renegade' and a 'corrupter of youth'"* (Dauben 1983, p. 122). Cantor replied with seriousness and great strength of mind to each of the criticisms that he received and in doing so he further refined his transfinite set theory.

Cantor was very keen on how his mathematical work could improve philosophy and theology and he made sure that his transfinite set theory was not at odds with the teachings of the Catholic Church. At this end, and because of the strong relation that Cantor felt between his mathematical work and his Christian beliefs, Cantor had an extensive correspondence with Catholic theologians and clerics in which he explained in great details his transfinite set theory and asked them to confirm that his idea that there are different kinds of infinite and not just one single infinite was not in disagreement with the Church doctrine (Dauben 1977, p. 95). The German philosopher Constantin Gutberlet (1837 – 1928) supported Cantor's theory and argued that the existence of Cantor's transfinite numbers was insured by God and that transfinite numbers were a reality because *"in the absolute [infinite] mind [of God] the entire sequence [of numbers, namely transfinite numbers] is always in actual consciousness, without any possibility of increase in the knowledge or contemplation of a new member of the sequence*" (Gutberlet 1886, p. 206).

Cantor believed that the reality of transfinite numbers, which he called *Transfinitum*, reflected God's infinity, an idea that was endorsed by Cardinal Johann Baptist Franzelin (1816 – 1886) who in his letter to Cantor on 26th January 1886 said that he observed *"with satisfaction how you [Cantor] distinguish very well the Absolute-Infinite and that which you call the Actual Infinite in the created. Because you explicitly declare the latter to be a "yet increasable" (naturally in indefinitum, that is, without ever being able to become a not more increasable) and set it against the Absolute as "essentially unincreasable," which obviously must be just as valid of the possibility and impossibility of reduction or subtraction; thus the two concepts of the Absolute-Infinite and the Actual-Infinite in the created, or Transfinitum, are essentially different, so that when both are compared, only the one must be characterized as genuine Infinite, the other as non-genuine and equivocal Infinite. Perceived thus, as far as I see until now, no danger for religious truths lies in your concept of the Transfinite"* (Cantor 1994, p.103)

Glorifying God through mathematics: Kepler, Euler, Cauchy

The relation between mathematics and the Christian faith certainly takes many forms and has many facets, but it also boils down to two types of approaches:

1) accepting and embracing it;

2) rejecting it altogether.

For example, the Scottish philosopher David Hume (1711 – 1776) in his treatise *An Enquiry on Human Understanding* published in 1748, expressed his firm conviction that reason alone is unable to build a convincing argument for the veracity of the Christian faith. Hume continued by affirming that *"whoever is moved by Faith to assent to it [Christian religion], is conscious of a continued miracle in his own person, which subverts all the principles of his understanding, and gives him a determination to believe what is most contrary to custom and experience"* (Hume 1825, p. 132). Hume's ideas concerning the relation between sciences, mathematics and faith were not shared by some of the greatest mathematicians of the time who thought that God gifted them with mathematical minds for His glory. They believed that their mathematical insights and discoveries were nothing but ways for the praise of God and for shedding some light on His mysteries because *"the fear of the LORD is the beginning of wisdom, and the knowledge of the Holy One is insight"* (Prov. 9: 10). For example, the Swiss mathematician Johann Bernoulli (1667 – 1748) saw mathematics as a means to glorify God and to understand Him and His attributes because *"nowhere is God's power and wisdom more evident than in the study of his works, and none is better equipped for this study than the philosopher and mathematician, who tries to fathom both the nature and character of God's works"* (Sierksman 1992, p. 28). Another example is the Scottish mathematician Colin Maclaurin (1698 – 1746) who considered God to be *"the Author and Governor of the universe"* (Turnbull 1951, p. 7) and considered the cosmos as the proclaimer of His handiwork. Similar to Bernoulli, Maclaurin saw in the study of mathematics the most suitable means for understanding God and His works.

Johannes Kepler

The German mathematician and astronomer Johannes Kepler (1571 – 1630) was a Lutheran who believed that mathematics was a rational tool that God gave us to help us with our understanding of Him and His cosmic plan. Kepler was convinced that God *"pushed into the light of knowledge the utilisation of the numbers, weights, and sizes which He marked out at creation. For these secrets are not of the kind whose research should be forbidden; rather they are set before our eyes like a mirror so that by examining them we observe to some extent the goodness and wisdom of the Creator"* (Caspar 1959, p. 381).

Kepler embraced Pythagoreans and Platonic views about the mathematical nature of the cosmos, which he expressed in the preface of his treatise *Mysterium Cosmographicum* (The Mystery of the Cosmos) published in 1596, where he stated that God *"in creating the Universe and regulating the order of the cosmos, had in view the five regular bodies of geometry known since the days of Pythagoras and Plato, and that the has fixed according to those dimensions, the number of heavens, their proportions and the relations of their movements"* (Kepler 1963, p. 15).
In particular, Kepler believed that God created the physical world according to a mathematical plan, which was at the same time simple and beautiful. He agreed with the Pythagoreans that the harmonies present in the mind of God were responsible for the planetary orbits to have the size, periodicity, and number that they have. In agreement with Platonists, Kepler affirmed that the primary causes in creation were to be attributed solely to the mathematical archetypes (Bradley 2011, p. 13). Hence, when we engage with mathematics, we share God's thoughts because we, as a collective, speak the same universal language with God. This means that through the application of a very rigorous mathematical methodology we have the possibility to unravel God's plan in the universe (Bradley 2011, p. 14).

Leonhard Euler

The Swiss mathematician Leonhard Euler (1707 – 1783), a pupil of Johann Bernoulli, was a Calvinist who declared that *"the principal aim of [the] knowledge [of truth] is God and His works, since all other truths to which reflection can lead mankind end with the Supreme Being and His works. For God is the truth, and the world is the work of His almightiness and His infinite wisdom. Thus, the more man learns to know God and His works, the further he will advance in the knowledge of the truth, which contributes just as much to the perfection of his understanding"* (Euler 1960, II). The more Euler was deepening its knowledge of mathematics, the more he believed that God was constantly creating and sustaining the universe. In fact, he asserted that the observation of the movements of the heavenly bodies through the centuries clearly showed a change in their orbits around the Sun and concluded that *"this provides an incontestable proof that the present structure of the world cannot be eternal, but it must have been produced at a particular time by immediate intervention of God"* (Euler 1960, LI).

Augustin-Louis Cauchy

The French mathematician Augustin-Louis Cauchy (1789 – 1857) was a devout Roman Catholic who defended and practiced his faith in his life and his work. He thought that *"the Christian religion is [...] highly favourable to the advancement of the sciences and to development of the most noble faculties of our intelligence"* (Belhoste 1991, p. 216). Cauchy not only thought that God could be glorified by means of his mathematical achievements, but also believed that his faith enlightened his scientific work and that, conversely, his mathematical attainments strengthened his Christian belief. Cauchy also affirmed that *"in many instances the science of numbers and analytical methods can help us to discover the truth or, at the very least, to recognise it"* (Belhoste 1991, p. 219).

Mathematics and 'the proof' of the existence of God: Descartes, Maupertuis

Since the Middle Ages, theologians, philosophers and scientists have tried to prove the existence of God by means of *rational proofs*, which were based on mathematical concepts and methods.

René Descartes

In 1630, the French philosopher and mathematician René Descartes (1595 – 1650) told the Minim Friar Marin Mersenne[13] that *"the mathematical truths, which you call eternal, have been established by God and depend on him entirely, just as all other creatures do. [...] He has established these laws in nature as a king establishes laws in his kingdom"* (Descartes 2010, pp. 259 – 260).

Descartes devoted his life to unravel the mysteries of the universe by means of reason alone. He even designed an ontological proof of the existence of God because "*nothing can be too remote to be reached eventually, or too well hidden to be discovered - just as long as we refrain from accepting as true anything that is not"*[14] (Descartes 1978, Book 2, p.51). This affirmation was based on Descartes' belief that *"God has given each of us a light to distinguish truth from falsehood"* (Descartes 1978, Book 3, p.60). Descartes based all of his enquiries on a method of doubt and used skeptical arguments to prove his hypotheses. Only if rigorous mathematical, geometric, and ontological proofs showed that something is *obviously true* (evident) then we can conclude that it is *utterly* *certain*. For example, Descartes found that the *"idea of a perfect being included existence in the same way as […] the idea of a triangle includes the equality of its three angles to two right triangles"* and from here he concluded that *"the existence of this perfect being, God, is at least as certain as any geometrical proof"* (Descartes 1978, Book 4, p. 72).

Descartes affirmed that God established certain physical and mathematical laws in nature and that through our power of reasoning we can be certain that these laws are obeyed everywhere in the universe. In his Fifth Meditation, Descartes asserts that he finds in his mind an idea of God as much as he finds an idea of a number or a geometric shape. He continues saying that his understanding of God's existence is as clear as his understanding of the proof of the properties of any number or shape. Hence, Descartes thinks that he *"should attribute to God's existence at least the same degree of certainty that I have attributed to mathematical truths until now"* (Descartes 2021, p. 168). However, based on his belief that if something is evident then it is entirely true, Descartes concludes that his knowledge of God and His attributes has led him to acquire a total knowledge of and certainty about many things *"both about God and other intellectual things, and also about as much of physical nature as falls within the scope of pure mathematics"* (Descartes 2021, p. 173). Thus, for Descartes mathematical reasoning is a way to achieve a certain understanding of everything in the world because God exists and He does not deceive, and so we can *see* the eternal truths as they truly are (Miller 1957, p. 461).

Pierre Louis Moreau de Maupertuis

The French mathematician and philosopher Pierre Louis Moreau de Maupertuis (1698 – 1759) constantly combined theological considerations to his scientific reflections and believed that a study of the wonders of the universe could lead to a proof of the existence of God (Maupertuis 1746, p. 268). For Maupertuis, the proofs of the existence of God are to be found in the general laws of nature because they show how motion is conserved and used on the basis of the attributes of a Supreme Intelligence. Hence, Maupertuis concludes that it is possible to make a better use of mathematics by diverting it from *"the trivial necessities of the body or the futile speculations of the spirit"* (Maupertuis 1746, p. 277) so that it can be used to search for new proofs of the existence of God (Panza 1995, p. 478). In fact, Maupertuis believed that *"the proofs of* *the existence of God [that mathematics] will provide should have over all others the advantage of being evident, which is typical of mathematical truths"* (Maupertuis 1746, 278).

In 1744, Maupertuis formulated the *Principle of Least Action* (PLA)[15] and because he thought that *"the Supreme Being [God] is everywhere, but is not everywhere visible to the same extent [then] we shall better see Him in the simplest objects. Let us search for Him in the first laws He imposed on nature […] according to which movement is conserved, distributed, or destroyed"* (Maupertuis 1746, 279). Since the PLA is a universal law from which specific laws can be deduced, then it could provide a proof of the existence of God because it is *"so wise a principle, and so worthy of the Supreme Being"* (Maupertuis 1746, 286).

Transcendence and mathematics in contemporary mathematics education

Mathematics is a sort of island separated from many other disciplines, and if there is any crossing is because mathematical tools are used in different contexts, and not because of a true synergy between disciplines.

In general, the mathematics curriculum in universities and colleges is divided into two areas:

- *Applied mathematics*: mathematical tools and ideas that find applications in physics, chemistry, biology, engineering, computer sciences, social sciences, economics, etc.

- *Pure mathematics*: abstract mathematical concepts and theories that do not have any obvious practical application.

If there is some contact between mathematics and the sciences, there is little relation between mathematics and the humanities. University and college courses may offer History of Mathematics and Philosophy of Mathematics modules, but there are no modules that cover the transcendent aspects of mathematics and its strong links with theology and faith. This is possibly due to the advent of positivism and of the scientific method in the 18th century and the fact that positivistic philosophies dominate the scientific landscape including the mathematical community. This does not mean that mathematicians are not religious, some of them are but they think that religion and mathematics have nothing to do with one another. In other instances, mathematicians are simply not interested in exploring the relation between mathematics and theology (Davis 2004, p. 2).

As we have seen in the previous sections, mathematics and theology/faith are intrinsically intertwined because the abstractedness of mathematical concepts is a consequence of the fact that numbers and their relations are in the mind of God. Hence, it would be important that the mathematics curriculum covered the transcendence of mathematics and its relation with faith and ultimately with God. Catholic universities and colleges could offer the ideal medium for the development of a truly cross-disciplinary mathematics curriculum where not only the mathematical works of the great mathematicians who recognised the presence of God in their works is explored, but the students could also be encouraged to continue in the footsteps of such leading mathematicians and investigate further the relation between transcendence and mathematics.

Conclusions

Since ancient times, mathematics has been considered a universal language, which God used to write the book of nature. Some of the greatest mathematicians from the 15^{th} century to the present day have seen their mathematical work as a way to better grasp the mystery of the existence of God and in particular of His infinity, as well as to glorify His name. The 20^{th} century has seen a drastic secularisation of mathematics and its drifting in the realm of high abstractedness, which led to a detachment from the sciences, philosophy and theology. In order to preserve the transcendent nature of mathematics, it would be important to include its exploration, as well as the relation between mathematics and theology in the mathematics curricula of Catholic universities and colleges.

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Notes

[1] Infinitesimal calculus is a mathematical discipline, which encompasses the calculation of limits, derivatives, integrals, and infinite series of mathematical functions. It was developed independently by the English mathematician and natural philosopher Sir Isaac Newton (1643 – 1727) and the German philosopher and mathematician Gottfried Wilhelm Leibniz (1646 – 1716).

[2] Complex numbers are numbers that can be expressed as $z = x + iy$ where x and y are the real and imaginary parts of the complex number z, respectively, and i is called the imaginary unity because $i = \sqrt{-1}$, which cannot be solved using real numbers.

[3] Differential geometry is a mathematical discipline that studies the geometry of mathematical curves and surfaces.

[4] The Enlightenment period, also called the "Age of Reason", spanned from the 18th century well into the 19th century. It saw an affirmation of the superiority of reason over human emotions and religious ideas, and led to the development of a rigorous scientific and philosophical discourse and method.

[5] Positivistic philosophies consider valid only that which can be either verified by using scientific methods or proved mathematically. Positivism rejects both metaphysics (philosophical system that deals with the first principles of things, such as being, knowing, time and space) and theism (the belief in the existence of a Divine Being that creates the world and intervenes in it).

[6] According to the Copenhagen interpretation, an object exists in the physical world only when someone observes it. For example, a mug exists in my reality only when I stare at it.

[7] Scriptures texts are from The New Revised Standard Version.

[8] Pythagoreanism is an ancient Greek philosophy based on the teaching of Pythagoras (ca. 570 – ca. 490 BC). Pythagoreans followed a highly structured lifestyle and exercised rigid self-discipline. They believed that numbers were fundamental to the understanding of the laws governing the cosmos, and also that world could be explained by means of mathematical proportions.

[9] In Platonism, the demiurge is a deity or divine craftsman, which fashions the physical world based on eternal ideas (forms).

[10] The first term of Bode's law is 0.4, which is very close to the actual distance of Mercury from the Sun, which is 0.39AU where AU indicate Astronomical Units (1 AU is equal to about 150 million kilometers or 93 million miles). The distances of the other planets from the Sun are obtained by means of Bode's law mathematical formula:

$$planet - sun\ distance = \sum_{n=0}^{6} \frac{(3 \cdot 2^{n}) + 4}{10}$$

which gives the numerical sequence 0.7, 1.0, 1.6, 2.8, 5.2, 10.0, 19.6. These values are quite close to the actual distances of the planets from the Sun, namely Venus (0.72AU), Earth (1.0AU), Mars (1.52AU), Asteroid Belt (2.8AU), Jupiter (5.2AU), Saturn (9.56 AU), and Uranus (19.8 AU).

[11] Dialogue on the connection between words and things.

[12] Transfinite set theory was created by Cantor to clarify mathematically what infinity is. Cantor showed with outstanding mathematical rigour that there is not just one type of infinity, but there is a hierarchy of infinites with some infinite being "larger" than others. This multiplicity of infinites allowed for comparison between collections of infinite numbers (namely sets containing an unthinkably large – infinite – number of numbers). The comparison between different kinds of infinite was possible by means of transfinite numbers, namely infinite quantities.

[13] Father Marin Mersenne (1588 – 1648) was called *The Secretary of Learned Europe* because of his extensive correspondence with the greatest thinkers of the time. He acted as a translator and editor, and was instrumental in disseminating scientific information across Europe. He had a lifelong connection with René Descartes.

[14] Translations of Descartes' quotations from his *Discourse de la Méthode* and *Méditations Métaphysiques* are mine.

[15] The Principle of Least Action states that an object subject to a force that travels from a point A to another point B will choose the trajectory for which the corresponding action is the minimum possible, where the action defines the motion of the object. PLA is widely applied in mechanics.